Discussion: Why is Asset Demand Inelastic

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Summary of the Paper

- Classical theory: demand elasticity ($\frac{d \ln w}{d \ln P}$) is very high ($\approx$7000)
- Empirically: measured demand elasticity is very low ($\approx$2)
- This paper: What explains the divergence?
  - Decompose demand elasticity into two factors:
    1. price pass-through
    2. weight responsiveness (substitutability of assets)
  - Classical theory: both are large
    1. expected return $\mu = E\left[\frac{P_{t+1}}{P_t}\right]$, $\ln \mu = \ln E\left[P_{t+1}\right] - \ln P_t$.

       Assuming P_{t+1} is constant gives $-\frac{d \ln \mu}{d \ln P_t} = 1$
    2. assets are almost perfectly substitutable
  - Empirically: both are small
    1. P_t can predict P_{t+1}
    2. measured substitutability is low

Main Comment: Outline

- Important in understanding why demand is inelastic in the data but elastic in classical models
- Provide guidelines on what types of models that can produce inelastic demand
- Comment: Decomposition in general includes many other factors
  - Discuss other factors

Summary of the Paper: Model

- Optimal portfolio holding under mean-variance utility

$$w = \frac{1}{\gamma A}\Sigma^{-1}\mu$$

  - expected return: $\mu_i = E\left[\frac{P_{i,t+1}}{P_{i,t}}\right]$; var-cov: $\Sigma_{i,j} = Cov\left(\frac{P_{i,t+1}}{P_{i,t}}, \frac{P_{j,t+1}}{P_{j,t}}\right)$
  - For simplicity, define $\rho_{i,j}$ as the (i,j)th element of Σ^{-1} which depends on ALL elements of Σ

- Rewrite portfolio weight ($\gamma = 1, A = 1$):

$$w_i = \sum_{j=1}^{n} \rho_{i,j}\mu_j$$

- Key of this paper: decompose the slope of demand curve

$$\frac{dw_i}{dP_{i,t}} = \frac{\partial w_i}{\partial \mu_i} \times \frac{\partial \mu_i}{\partial P_{i,t}} = \rho_{i,i}\frac{\partial \mu_i}{\partial P_{i,t}} \tag{1}$$

  - $\frac{\partial w_i}{\partial \mu_i}$: weight responsiveness; $\frac{\partial \mu_i}{\partial P_{i,t}}$: price pass-through

Summary of the Paper: Intuition

$$\frac{dw_i}{dP_{i,t}} = \rho_{i,i}\frac{\partial \mu_i}{\partial P_{i,t}}$$

- Classical model implies $\rho_{i,i} \approx \infty$, $\frac{\partial \mu_i}{\partial P_{i,t}} \approx 1$, gives elasticity of 7000.
- This paper's estimation: $\rho_{i,i} \approx \frac{1}{0.006}$, $\frac{\partial \mu_i}{\partial P_{i,t}} \approx 0.06$, elasticity of 10.
- Both weight responsiveness and price pass-through are small in data
  - assets are not perfectly substitutable
  - P_t has predictive power for P_{t+1}
- This explains the divergence between theory and data.

Main Comment: Assumptions for Decomposition

- Slope of demand curve is, in general (from $w_i = \sum_{j=1}^{n} \rho_{i,j}\mu_j$)

$$\frac{dw_i}{dP_{i,t}} = \sum_{j=1}^{n} \left[\rho_{i,j} \frac{\partial \mu_j}{\partial P_{i,t}} + \mu_j \frac{\partial \rho_{i,j}}{\partial P_{i,t}} \right] \tag{2}$$

  - i.e., change in $P_{i,t}$ can change all assets' expected returns and var-cov
- This paper's simple decomposition: $\frac{dw_i}{dP_{i,t}} = \rho_{i,i} \frac{\partial \mu_i}{\partial P_{i,t}}$
- Additional two assumptions need to be made:
  1. $\frac{\partial \mu_j}{\partial P_{i,t}} = 0$ for $i \neq j$
  2. $\frac{\partial \rho_{i,j}}{\partial P_{i,t}} = 0$ for all i, j
- Comment: Examine the assumptions under which conditions these other terms can be ignored.

Assumption 1

- **Assumption 1:** $\frac{\partial \mu_j}{\partial P_{i,t}} = 0$ **for** $i \neq j$**. i.e.,**

$$\frac{\partial}{\partial P_{i,t}} E\left[\frac{P_{j,t+1}}{P_{j,t}}\right] = 0 \tag{3}$$

- How can this assumption fail?
  - In general equilibrium, all prices are determined simultaneously. If one price changes, all other prices have to change to clear the market.
  - Learning from prices, e.g., Admati (1985). When $P_{i,t}$ changes, it affects the belief of $E\left[\frac{P_{j,t+1}}{P_{j,t}}\right]$ for all other j.
- However, the above considerations in a frictionless partial equilibrium economy, which this paper focuses on, will likely be small.
  - Therefore, in what follows, I assume $P_{i,t+1}$ and $P_{j,t+1}$ will not change when $P_{i,t}$ changes.

Main Comment: Assumption 2

- **Assumption 2:** $\frac{\partial \rho_{i,j}}{\partial P_{i,t}} = 0$ **for all** i, j**.**
- Comment: $\frac{\partial \rho_{i,j}}{\partial P_{i,t}}$ has a first-order impact so that cannot be ignored — I call it "Var-Cov pass-through".
- This paper: price pass-through $\frac{\partial \mu_i}{\partial P_{i,t}}$ has a first-order impact on demand elasticity

$$\frac{\partial \mu_i}{\partial P_{i,t}} \equiv \lim_{\Delta \to 0} \frac{1}{\Delta} E\left[\frac{P_{i,t+1}}{P_{i,t}+\Delta} - \frac{P_{i,t+1}}{P_{i,t}}\right],$$

  - Use the following approx: $\frac{P_{i,t+1}}{P_{i,t}+\Delta} - \frac{P_{i,t+1}}{P_{i,t}} \approx -\frac{P_{i,t+1}}{P_{i,t}^2}\Delta$,

$$\frac{\partial \mu_i}{\partial P_{i,t}} = -\frac{\mu_i}{P_{i,t}} \text{ does not converge to 0 when } \Delta \to 0.$$

- Question: how does "Var-Cov pass-through" $\frac{\partial \rho_{i,j}}{\partial P_{i,t}}$ behave?

Main Comment: Assumption 2

- Recall $\rho_{i,j} \equiv \Sigma_{i,j}^{-1}$, i.e., "Var-Cov pass-through" $\frac{\partial \rho_{i,j}}{\partial P_{i,t}}$ has the same order of impact as

$$\frac{\partial \Sigma_{i,j}}{\partial P_{i,t}} = \frac{\partial}{\partial P_{i,t}} Cov\left(\frac{P_{i,t+1}}{P_{i,t}}, \frac{P_{j,t+1}}{P_{j,t}}\right)$$

- For $i \neq j$ (let me call it "Cov pass-through"):

$$\begin{aligned} \frac{\partial \Sigma_{i,j}}{\partial P_{i,t}} &\equiv \lim_{\Delta \to 0} \frac{1}{\Delta}\left[Cov\left(\frac{P_{i,t+1}}{P_{i,t}+\Delta}, \frac{P_{j,t+1}}{P_{j,t}}\right) - Cov\left(\frac{P_{i,t+1}}{P_{i,t}}, \frac{P_{j,t+1}}{P_{j,t}}\right)\right] \\ &\approx \lim_{\Delta \to 0} \frac{1}{\Delta} Cov\left(-\frac{P_{i,t+1}}{P_{i,t}^2}\Delta, \frac{P_{j,t+1}}{P_{j,t}}\right) = -\frac{\Sigma_{i,j}}{P_{i,t}} \end{aligned}$$

- also has a first-order impact on demand elasticity (just as price pass-through $\frac{\partial \mu_i}{\partial P_{i,t}}$)!!

Main Comment: Assumption 2

- For $i = j$ (let me call it "Var pass-through"):

$$\frac{\partial \Sigma_{i,i}}{\partial P_{i,t}} \equiv \lim_{\Delta \to 0} \frac{1}{\Delta} \left[Cov\left(\frac{P_{i,t+1}}{P_{i,t} + \Delta}, \frac{P_{i,t+1}}{P_{i,t} + \Delta} \right) - Cov\left(\frac{P_{i,t+1}}{P_{i,t}}, \frac{P_{i,t+1}}{P_{i,t}} \right) \right]$$
$$= -2 \frac{\Sigma_{i,i}}{P_{i,t}}$$

- This is also a first-order impact on demand elasticity!

Suggestions

- Comment:
  - What are the sign and magnitude of “Var-Cov pass through”?
- Suggestions:
  - elaborate more on why you ignore “Var-Cov pass through”
  - perhaps provide a bound on the magnitude of those ignored terms
  - or explicitly estimate the ignored terms in the data

Conclusion

- Extremely interesting paper
- Important in understanding why demand is inelastic in the data but elastic in classical models
- Guidelines on what types of models that produce inelastic demand
  - Models with substitutable assets
  - Models with price reversal/momentum
- Elaborate clearly on model assumptions and estimate missing factors