Discussion: Investor Beliefs and Asset Prices Under Selective Memory

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Summary of the Paper

- **Big Picture**
  - Subjective expectations (from survey data) differ from objective expectations (estimated by econometricians using realized data).
- **Key Idea**
  - A representative investor may exhibit behavioral biases in belief formation.
  - A consumption-based asset pricing model with similarity-weighted memory helps explain the divergence between subjective and objective expectations.
- **Main Findings**
  - Subjective beliefs overreact to new information.
  - Time-varying subjective risk premium.
    - Subjective expected returns are procyclical and not predictable by the p/d ratio.
    - Objective expected returns are countercyclical and predictable by the p/d ratio.

Main Comments: Outline

- Interesting application of similarity-weighted memory theory to asset pricing models.
- **Comment 1**: Identify unique implications of the model compared to existing literature.
  - For example, Nagel and Xu (2022) show that faded memory leads to similar implications as those in this paper.
- **Comment 2**: Clarify and replicate empirical evidence.
  - The paper currently cites empirical evidence from different sources using different data, sample periods and measurements.

Summary of Similarity-Weighted Memory Model

- Assume the rep investor selectively recalls past data that resembles the current data.
- **Log endowment growth**:

$$g_t = \mu_t + \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1)$$

  Two-state Markov chain: $\mu_H > \mu_L$ and $\sigma_H < \sigma_L$. State H: good times, state L: bad times.
- The investor learns about the distribution of g_t from recalled history g_τ ($\tau = -\infty \to t$).
- **Distorted memory function**:

$$m(g_\tau) = \exp\left[-\frac{(g_\tau - g_t)^2}{2\kappa}\right]$$

- Assigns higher probability to past observations g_τ that are close to the current g_t.
- **Implication 1 of this paper**: Investors overweight current data, underweight history (priors), leading to overreaction to new information.

Comment 1: Clarifying Empirical Evidence and Theory

- Empirical evidence is drawn from cited literature, but some citations are not precise.
  - Cited evidence: Coibion and Gorodnichenko (2015) document **underreaction** in consensus forecasts.
  - Correct citation: Bordalo et al (2020): **Overreaction** in individual forecasts.
  - This paper: A rep agent model, which can explain overreaction at the individual level but doesn't have heterogeneous agents to address consensus beliefs.
- Alternative Interpretations of overreaction to news:
  - Behavioral Models:
    - Bordalo et al (2020): Diagnostic expectations.
    - Nagel and Xu (2022): Faded memory.
  - Rational Models:
    - Li, Van Nieuwerburgh, Renxuan (2024): Slow learning about the long-run mean.
    - Han (2024): heterogeneous agents with asymmetric information.
- **Comment:** Clarify the citations and compare your theory with existing alternatives.

Comment 1: Clarifying Empirical Evidence and Theory

- Nagel and Xu (2023): show that objective expected returns are countercyclical, while subjective expected returns are acyclical.
- NX regress realized returns on business cycle indicators, coefficient > 0.

N-IP		TERM		DEFAULT		F1		VIX^2	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
1.74	1.53	4.13	4.30	3.32	3.24	4.24	3.92	1.49	1.22
{1.55}	{1.26}	{4.40}	{4.62}	{2.84}	{2.51}	{4.54}	{4.16}	{1.82}	{1.48}

- NX regress survey-forecasted returns on business cycle indicators, coefficient $= 0$.

N-IP		TERM		DEFAULT		F1		VIX^2	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.98	1.57	0.44	0.46	-0.30	0.57	0.15	0.63	-0.01	0.27
(0.17)	(0.01)	(0.20)	(0.18)	(0.35)	(0.13)	(0.68)	(0.08)	(0.95)	(0.14)

- **This paper:** Claims subjective expected returns are procyclical. Evidence?

Comment 1: Clarifying Empirical Evidence and Theory

- This paper's simulation results:

	No parameter uncertainty			Parameter uncertainty		
Symbol	Total	Normal	Recession	Total	Normal	Recession
Subjective asset prices						
$\overline{er_t}$	3.385	3.386	3.381	3.288	3.256	3.480
Std (er_t)	0.009	0.001	0.021	0.902	2.213	3.474
corr (er_t, g_t)	0.999	1.000	0.999	0.419	0.010	0.004
$\overline{r_t^f}$	2.186	2.187	2.178	2.174	2.180	2.136
Std(r_t^f)	0.027	0.012	0.065	0.686	0.848	0.902
corr(r_t^f, g_t)	0.999	1.000	0.999	0.497	0.069	0.326
$\overline{rp_t}$	1.199	1.199	1.204	1.114	1.076	1.344
Std(rp_t)	0.005	0.002	0.002	1.217	2.410	3.660
corr(rp_t, g_t)	−0.990	−1.000	−0.990	−0.381	−0.005	−0.057
Objective asset prices						
$\overline{rp_t}$	0.998	0.776	2.360	3.209	1.768	12.054
Std(rp_t)	2.237	1.027	5.397	26.864	42.188	69.590
corr(rp_t, g_t)	−1.000	−1.000	−1.000	−0.348	−0.112	−0.399

- Objective expected excess return and endowment growth: correlation = -0.3 (countercyclical).
- Subjective expected return and endowment growth: correlation = 0.4 (procyclical).

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- However, the procyclicality arises from the risk-free rate. Subjective expected excess returns are still countercyclical!
- **Comment**: Compute the same moments in the data and compare them with the model. Otherwise, it's hard to assess quantitative success.

Comment 2: Unique Implications

- **Implication 2 of this paper**: Time-varying subjective risk premium.

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- However, Nagel and Xu (2022) also show time-varying subjective risk premiums, though very small, due to Jensen's inequality.
- The quantitative magnitude in this paper is also very small.

Comment 2: Unique Implications

- Potential unique implication: Return extrapolation (Greenwood and Shleifer (2014)).
  - Regress subjective risk premium on realized return — coefficient is significantly positive.
  - This paper claims success in generating this predictability to distinguish from Nagel and Xu (2022) but does not provide details to verify the claim.
- **Comment**: Replicate the GS regression in the model and compare it with the data. Otherwise, it's difficult to argue for the model's unique implications.

Conclusion

- An interesting application of similarity-weighted memory from Fudenberg, Lanzani, and Strack (2023).
- Requires clear empirical evidence to support claims of quantitative success.
- Needs to demonstrate unique implications compared to existing literature.