Online Appendix to
The Cross Section of the Monetary Policy Announcement Premium

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Appendix IA. Additional tables

In this section, we provide additional empirical evidence for the implied variance reduction, the portfolio sorting in Section 2,and other risk factors in Section 3. Table IA.1 demonstrates that our EVR is the best predictor of the actual IV reduction compared with four other alternative regression specifications. The results in Table IA.1 show that other potential predictors not only have a less significant coefficient, but also generate a lower R^2 than the EVR does in Eq.(2).Table IA.2 shows that the portfolio sorting results in Table 2 remain robust after we exclude the recent financial crisis period from July 2008 to June 2009, or after we exclude those firms with earnings announcements released on FOMC announcement days. Since VIX significantly increases before FOMC announcements and drops afterwards, Table IA.3 investigates our portfolio sorting results after controlling for the change of VIX around FOMC announcement days. Our findings on the EVR-sorted portfolios and CAPM beta-sorted portfolios remain largely unchanged. Table IA.4 presents the Fama-MacBeth regression and the pooled regression results when using two alternative co-skewness and co-kurtosis measures following Ang, Chen, and Xing (2006) and Guidolin and Timmermann (2008). We show that the risk price of the EVR remains robust after controlling for the co-skewness and co-kurtosis risk.

Table IA.1: Firm-level implied variance reduction forecast (alternative models)

	Actual IV Reduction				
	EVR	Model 1	Model 2	Model 3	Model 4
Predicted IV Reduction	0.2164	0.0402	0.0252	0.0219	0.0002
	(9.57)	(1.63)	(0.83)	(3.69)	(0.15)
Constant	0.0021	-0.0019	-0.0019	0.0024	0.0024
	(0.91)	(-0.83)	(-0.83)	(1.13)	(1.13)
$R^2(\%)$	6.3	0.01	0.01	0.30	0.01

This table compares the forecast results of our EVR measure with other candidate measures when regressing the actual implied variance (IV) reduction on various IV reduction forecasts. In Model 1 (2) we use the median (mean) of IV reduction on previous FOMC days during the past twelve months as the IV reduction forecast for the upcoming FOMC announcement. In Model 3 (4) we adjust IV by the median (mean) historical realized variance in a similar manner as EVR to calculate the IV reduction forecast. The t-statistics using the day-clustered standard error are in parenthesis.

Table IA.2: Portfolio returns sorted on expected sensitivity (robustness check)

Panel A: Exclude the financial crisis

	1	2	$3-8$	9	10	$(10-1)$
$R^M - r_f$	1.44	1.19	0.93	1.24	1.44	0.00
$(R^M - r_f)\times$ FOMC Dummy	-0.05	-0.05	0.00	-0.07	0.10	0.15
	(-0.49)	(-0.77)	(-0.05)	(-1.26)	(0.78)	(0.78)
Non-FOMC Dummy	-0.80	-0.33	0.47	0.21	-1.68	-0.87
	(-0.70)	(-0.37)	(1.65)	(0.28)	(-1.44)	(-0.68)
FOMC Dummy	-13.21	-8.09	0.24	9.45	11.90	25.11
	(-1.81)	(-1.66)	(0.16)	(2.06)	(1.58)	(2.86)

Panel B: Exclude firms' earnings announcements

	1	2	$3-8$	9	10	$(10-1)$
$R^M - r_f$	1.42	1.15	0.90	1.17	1.33	-0.09
$(R^M - r_f)\times$ FOMC Dummy	0.04	-0.02	-0.01	-0.05	0.12	0.08
	(0.39)	(-0.42)	(-0.81)	(-0.99)	(1.29)	(0.48)
Non-FOMC Dummy	-1.29	-1.06	0.81	-0.33	-0.59	0.70
	(-0.97)	(-1.21)	(2.86)	(-0.42)	(-0.49)	(0.47)
FOMC Dummy	-13.33	-6.78	-0.91	6.92	12.06	25.39
	(-1.61)	(-1.56)	(-0.61)	(1.72)	(1.62)	(2.63)

This table reports the robustness check of our portfolio sorting results, as shown in Table 2. We report the daily regression results of Eq.(3), which include a non-FOMC dummy, an FOMC dummy, the market excess return, and its interaction with the FOMC dummy, as well as the Newey-West t-statistics. In Panel A, we exclude the height of the recent financial crisis from July 2008 to June 2009; in Panel B, we exclude those firms reporting earnings on the upcoming FOMC announcement days when we form portfolios. We use the COMPUSTAT variable "RDQ" as the reported date of quarterly earnings. The number of firms reporting earnings on FOMC days in our sample changes from 1 to 123. On average there are 25 firms reporting earnings on each FOMC announcement day.

Table IA.3: CAPM of portfolio returns controlling for VIX change

Panel A: Portfolios sorted on expected sensitivity

	1	2	$3-8$	9	10	$(10-1)$
$R^M - r_f$	1.52	1.23	0.90	1.26	1.55	0.03
$(R^M - r_f)\times$ FOMC Dummy	0.08	-0.03	-0.02	-0.07	0.13	0.05
	(0.71)	(-0.63)	(-1.02)	(-1.40)	(0.94)	(0.25)
Non-FOMC Dummy	-1.11	-0.51	0.63	0.41	-1.35	-0.24
	(-0.97)	(-0.58)	(2.20)	(0.55)	(-1.15)	(-0.19)
FOMC Dummy	-11.43	-7.42	-0.81	9.70	17.20	28.63
	(-1.59)	(-1.52)	(-0.51)	(2.13)	(2.18)	(3.14)

Panel B: Portfolios sorted on CAPM beta

	1	2	$3-8$	9	10	$(10-1)$
$R^M - r_f$	0.42	0.60	1.01	1.67	1.94	1.52
$(R^M - r_f)\times$ FOMC Dummy	0.04	-0.01	-0.02	0.02	0.24	0.20
	(1.24)	(-0.26)	(-1.38)	(0.25)	(2.76)	(1.82)
Non-FOMC Dummy	1.33	1.49	0.37	-2.27	-1.59	-2.93
	(2.02)	(2.40)	(1.05)	(-1.98)	(-0.95)	(-1.40)
FOMC Dummy	-4.48	-5.34	-2.00	1.27	6.81	11.29
	(-1.05)	(-1.54)	(-1.27)	(0.21)	(0.80)	(0.97)

This table reports the regression coefficients on a non-FOMC dummy, an FOMC dummy, the market excess return, and its interaction with the FOMC dummy, as well as the Newey-West t-statistics in the following regression:

$$R_t^i - r_{f,t} = \alpha_{Non}^i \cdot \mathbf{1}_{Non} + \alpha_{FOMC}^i \cdot \mathbf{1}_{FOMC} + \beta^i \left(R_t^M - r_{f,t}\right) + \beta_{FOMC}^i \left(R_t^M - r_{f,t}\right) \cdot \mathbf{1}_{FOMC} + \gamma \Delta VIX_t^2 + \varepsilon_t^i,$$

where R_t^i is the daily return of portfolio i, R_t^M is the daily return of the market, and $r_{f,t}$ is the daily risk-free rate. ΔVIX_t^2 is the daily change in VIX^2. $\mathbf{1}_{Non}$ and $\mathbf{1}_{FOMC}$ are dummy variables that take a value of 1 only on non-FOMC and FOMC announcement days, respectively. Panel A and B reports the results from EVR-sorted portfolios and CAPM beta-sorted portfolios, respectively.

Table IA.4: Other measures of co-skewness and co-kurtosis Risk

Panel A: Fama-Macbeth regressions on FOMC announcement days

	(1)	(2) CoSkew1	(3) CoSkew2	(4)CoKurt1	(5) CoKurt2	(6) CoSkew1 & CoKurt1	(7) CoSkew2 & CoKurt2
Constant	0.40	0.01	0.01	0.00	0.08	0.01	0.07
	(3.11)	(0.11)	(0.26)	(0.10)	(1.37)	(0.10)	(1.26)
EVR	0.20	0.18	0.18	0.19	0.20	0.19	0.20
	(2.52)	(3.10)	(3.12)	(3.19)	(3.48)	(3.31)	(3.53)
$\widehat{\beta}_{Mkt}$		0.36	0.36	0.44	0.36	0.45	0.36
		(2.73)	(2.71)	(2.40)	(2.55)	(2.52)	(2.58)
$\widehat{\beta}_{CoSkew}$		0.03	-0.08			0.04	-0.00
		(0.49)	(-0.71)			(0.69)	(-0.04)
$\widehat{\beta}_{CoKurt}$				-0.01	-0.03	-0.03	-0.03
				(-0.31)	(-0.99)	(-0.76)	(-0.87)
$R^2(\%)$	0.49	5.63	5.72	5.55	6.35	6.08	6.78

Panel B: Pooled regression

	$\mathbf{1}_{Non}$	$\mathbf{1}_{FOMC}$	EVR	$EVR \cdot \mathbf{1}_{FOMC}$	$\widehat{\beta}_{CoSkew}$	$\widehat{\beta}_{CoSkew} \cdot \mathbf{1}_{FOMC}$	$\widehat{\beta}_{CoKurt}$	$\widehat{\beta}_{CoKurt} \cdot \mathbf{1}_{FOMC}$	$R^2(\%)$
$CoSkew1$	0.02	0.41	0.06	0.14	-0.00	-0.06			0.10
	(1.07)	(3.64)	(2.51)	(1.39)	(-0.01)	(-1.41)			
$CoSkew2$	0.02	0.45	0.06	0.14	0.02	0.24			0.10
	(1.01)	(3.82)	(2.50)	(1.39)	(0.19)	(0.47)			
$CoKurt1$	0.01	0.27	0.06	0.15			0.00	0.01	0.10
	(0.76)	(2.36)	(2.51)	(1.54)			(0.55)	(2.36)	
$CoKurt2$	0.03	0.27	0.06	0.14			-0.00	0.07	0.10
	(0.73)	(1.28)	(2.50)	(1.40)			(-0.17)	(0.69)	
$CoSkew1 + CoKurt1$	0.01	0.26	0.06	0.15	-0.00	-0.06	0.00	0.01	0.10
	(0.76)	(2.30)	(2.51)	(1.55)	(-0.03)	(-1.32)	(0.55)	(2.37)	
$CoSkew2 + CoKurt2$	0.03	0.25	0.06	0.14	0.01	0.46	-0.00	0.10	0.10
	(0.72)	(1.24)	(2.50)	(1.43)	(0.12)	(0.88)	(-0.11)	(0.95)	

The table reports regression results of daily excess stock returns controlling for alternative measures of co-skewness risk and co-kurtosis risk. For the Fama-MacBeth regression in Panel A, we first compute co-skewness and co-kurtosis of each firm using the past 12-month daily excess returns two days before each FOMC announcement. Then on each FOMC announcement day, we regress the cross-sectional stock excess returns on EVR and co-skewness and (or) co-kurtosis risks from the first step. We report the average loadings on EVR and other risk factors and their associated Newey-West t-statistics. Co-skewness and co-kurtosis are calculated using the approach in Ang, Chen, and Xing (2006) and Guidolin and Timmermann (2008). For the pooled regression in Panel B, we report the coefficients in the following regression:

$$
\begin{aligned}
R_t^j - r_{f,t} &= \gamma_0 \cdot \mathbf{1}_{Non} + \gamma_{0,FOMC} \cdot \mathbf{1}_{FOMC} + \gamma_1 \cdot EVR_t^j + \gamma_2 \cdot EVR_t^j \cdot \mathbf{1}_{FOMC} \\
&\quad + \kappa_1 \cdot \hat{\beta}^j_{Other,t} + \kappa_2 \cdot \hat{\beta}^j_{Other,t} \cdot \mathbf{1}_{FOMC} + \varepsilon_t^j.
\end{aligned}
$$

For each stock, $\widehat{\beta}^j_{Other,t}$ are co-skewness and co-kurtosis, and are estimated using daily returns during the past twelve months. t-statistics are calculated using the trading-day clustered standard errors. The unit of regression coefficients is in percentage. Our full sample period is from January 1996 to December 2017 with 176 FOMC days.

Appendix IB. Additional details of the continuous-time model

IB.1. Pre-announcement information release

In this section, we show the solution to a model where information releases at time $\tau < T$ before the actual announcements. In this model, investors observe a pre-announcement signal of Eq.(26). We first present the dynamics of posterior beliefs and then solve for the price-to-dividend ratio which is used to generate the pre-announcement drift.

In the interior of $(0, \tau)$, the posterior mean and variance of θ_t follow Eq.(11) and (12). $q(t)$ has the closed-form solution given in Eq.(B.8). At time τ, investors start to receive an additional signal ζ_t. The following lemma summarizes the dynamics of posterior beliefs from τ to T.

Lemma 1. *In the period of $\tau \leq t < T$ when investors observe a pre-announcement signal, the posterior mean and variance of θ_t can be written as*

$$d\hat{\theta}_t = a\left(\mu - \hat{\theta}_t\right)dt + \frac{q_t}{\sigma}d\tilde{B}_{A,t} + \frac{q_t}{\sigma_\zeta}d\hat{B}_{\zeta,t}, \tag{IB.1}$$

$$dq_t = \left[\sigma_\theta^2 - 2aq_t - \left(\frac{1}{\sigma^2} + \frac{1}{\sigma_\zeta^2}\right)q_t^2\right]dt, \tag{IB.2}$$

where $d\hat{B}_{\zeta,t} = \frac{1}{\sigma_\zeta}\left(d\zeta_t - \hat{\theta}_t dt\right)$. The closed-form solution for $q(t)$ is

$$q(t) = \frac{\sigma_\theta^2\left(1 - ke^{-2\hat{b}(t-\tau)}\right)}{\left(\hat{b} - a\right)ke^{-2\hat{b}(t-\tau)} + a + \hat{b}}, \tag{IB.3}$$

where $\hat{b} = \sqrt{a^2 + \sigma_\theta^2\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_\zeta^2}\right)}$, $k = \frac{\sigma_\theta^2 - q_\tau\left(a+\hat{b}\right)}{\sigma_\theta^2 - q_\tau\left(a-\hat{b}\right)}$, and q_τ satisfies Eq.(B.8).

Proof. Given the general solution for the Riccati equation that characterizes $q(t)$, on $(0, \tau)$, q_t is given by (B.8), and on (τ, T), q_t satisfies (IB.3). The constant k in (IB.3) is chosen by the continuity of $q(t)$ at $t = \tau$. □

Following the same procedures as we show in Section B.3, the price-to-dividend ratio for $\tau < t < T$ satisfies the following PDE:

$$1 - p^i\left(\hat{\theta}_t, t\right)\varpi^i\left(\hat{\theta}_t, t\right) + p^i_t\left(\hat{\theta}_t, t\right) - p^i_\theta\left(\hat{\theta}_t, t\right)\nu^i\left(\hat{\theta}_t, t\right) + \frac{1}{2}p^i_{\theta\theta}\left(\hat{\theta}_t, t\right)\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_\zeta^2}\right)q_t^2 = 0, \tag{IB.4}$$

with the boundary condition defined in Eq.(B.24) and

$$\begin{aligned}
\varpi^i\left(\hat{\theta}_t, t\right) &= \rho - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma^2 + \gamma\sigma^2\eta_i + \frac{1}{\psi}\hat{\theta}_t - \mu - \xi_i\left(\hat{\theta}_t - \mu\right) \\
&\quad + \frac{\frac{1}{\psi} - \gamma}{a + \rho}q_t\left(1 - \eta_i\right) + \frac{1}{2}\frac{\left(\frac{1}{\psi} - \gamma\right)\left(1 - \frac{1}{\psi}\right)}{(a + \rho)^2}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_\zeta^2}\right)q_t^2, \\
\nu^i\left(\hat{\theta}_t, t\right) &= a\left(\hat{\theta}_t - \mu\right) + \left(\gamma - \eta_i\right)q_t - \frac{\frac{1}{\psi} - \gamma}{a + \rho}\left(\frac{1}{\sigma^2} + \frac{1}{\sigma_\zeta^2}\right)q_t^2.
\end{aligned}$$

IB.2. Numerical solutions

Here we describe the details of how to numerically solve the PDE of price-to-dividend ratio with the boundary condition for firm i, described in Lemma 3.[1] The Feynman-Kac formula implies that the solution could be written as the following conditional expectation problem:

$$p\left(\hat{\theta}_t, t\right) = \mathbb{E}\left[\int_t^T e^{-\int_t^s \varpi\left(\hat{\theta}_u, u\right)du}ds + e^{-\int_t^T \varpi\left(\hat{\theta}_u, u\right)du}p\left(\hat{\theta}_T, T\right)\right], \tag{IB.5}$$

where the state variable $\hat{\theta}_t$ follows the law of motion

$$d\hat{\theta}_t = -\nu\left(\hat{\theta}_t, t\right)dt + \frac{q_t}{\sigma}dB_t. \tag{IB.6}$$

[1] We fix (ξ_i, η_i) for each firm and calculate the firm-specific price-to-dividend $p^i\left(\hat{\theta}_t, t\right)$. Since the procedure is the same for each firm, we drop the superscript i for simplicity.

Note that we construct the above auxiliary problem to solve the PDE (B.23), where $\varpi\left(\hat{\theta}_t, t\right)$ could be interpreted as the discount rate, and $-\nu\left(\hat{\theta}_t, t\right)$ and $\frac{q_t}{\sigma}$ are the drift and diffusion process in the SDE of $\hat{\theta}_t$.

Now we need to solve (IB.5) and (IB.6) with the boundary condition defined in (20). The major steps are:

1. Start with an initial guess of a pre-announcement price-to-dividend ratio $p\left(\hat{\theta}_T, T\right) = 1/\rho$.

2. With the initial guess of $p\left(\hat{\theta}_T, T\right)$, for $t = T - \Delta$, $T - 2\Delta$, etc., we rewrite the discounted problem in (IB.5) in a recursive formula:

$$p\left(\hat{\theta}_t, t\right) = \Delta + e^{-\varpi\left(\hat{\theta}_t, t\right)\Delta}\mathbb{E}\left[p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)\right], \tag{IB.7}$$

   and compute the price-to-dividend ratio backwards (from T to 0) recursively until we obtain $p\left(\hat{\theta}_0, 0\right)$.

3. Compute an updated pre-announcement price-to-dividend ratio function, $p\left(\hat{\theta}_T, T\right)$ using Eq.(20),

$$p\left(\hat{\theta}_T, T\right) = \frac{\mathbb{E}\left[e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0} p\left(\hat{\theta}_0, 0\right) \mid \hat{\theta}_T, q_T\right]}{e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_T + \frac{(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)}{2(a+\rho)^2}(q_T - q_0)}}. \tag{IB.8}$$

4. Go back to step 1 and iterate until the function $p\left(\hat{\theta}_T, T\right)$ converges.

Now we discuss the numerical details. For simplicity, define the log price-to-dividend ratio as $\varrho\left(\hat{\theta}_t, t\right) \equiv \ln p\left(\hat{\theta}_t, t\right)$. Therefore, Eq.(IB.7) could be written as

$$\begin{aligned}
\varrho\left(\hat{\theta}_t, t\right) &= \ln\left\{\Delta + e^{-\varpi\left(\hat{\theta}_t, t\right)\Delta}\mathbb{E}\left[e^{\varrho\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)}\right]\right\} \\
&= \ln\left\{\Delta + e^{-\varpi\left(\hat{\theta}_t, t\right)\Delta}\mathbb{E}\left[e^{\varrho\left(\hat{\theta}_t, t+\Delta\right)+\varrho_\theta\left(\hat{\theta}_t, t\right)\left(\hat{\theta}_{t+\Delta}-\hat{\theta}_t\right)}\right]\right\} \\
&= \ln\left[\Delta + e^{-\varpi\left(\hat{\theta}_t, t\right)\Delta}e^{\varrho\left(\hat{\theta}_t, t+\Delta\right)-\varrho_\theta\left(\hat{\theta}_t, t\right)\nu\left(\hat{\theta}_t, t\right)\Delta+\frac{1}{2}\Delta\varrho_\theta^2\left(\hat{\theta}_t, t\right)\left(\frac{q_t}{\sigma}\right)^2}\right] \\
&= \ln\left[\Delta + e^{\varrho\left(\hat{\theta}_t, t+\Delta\right)-\left[\varpi\left(\hat{\theta}_t, t\right)+\varrho_\theta\left(\hat{\theta}_t, t\right)\nu\left(\hat{\theta}_t, t\right)-\frac{1}{2}\varrho_\theta^2\left(\hat{\theta}_t, t\right)\left(\frac{q_t}{\sigma}\right)^2\right]\Delta}\right],
\end{aligned}$$

where the second equivalence comes from the log linear approximation around $\hat{\theta}_t$ and the third equivalence uses the fact that $\hat{\theta}_t$ follows normal distribution and $\left(\hat{\theta}_{t+\Delta} - \hat{\theta}_t\right) \sim \mathcal{N}\left(-\nu\left(\hat{\theta}_t, t\right)\Delta, \left(\frac{q_t}{\sigma}\right)^2\Delta\right)$ from Eq.(IB.6).

At the announcement T, (IB.8) gives

$$\varrho\left(\hat{\theta}_T, T\right) = \ln\mathbb{E}\left[e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0, 0\right)} \mid \hat{\theta}_T, q_T\right] - \left[\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_T + \frac{(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)}{2\left(a+\rho\right)^2}\left(q_T - q_0\right)\right]. \tag{IB.9}$$

We approximate the above expectation in two ways.

First, if $\hat{\theta}_0$ is in the interior $\hat{\theta}_0 \in \left[\hat{\theta}_{min}, \hat{\theta}_{max}\right]$, we use the Gaussian quadrature. We generate a vector of nodes $\mathbf{n}$ and the corresponding weights ω to approximate the normal distribution $\mathcal{N}\left(0, q_T - q_0\right)$. Then $\hat{\theta}_0 = \hat{\theta}_T + \mathbf{n}$ approximates the normal distribution of $\hat{\theta}_0 \sim \mathcal{N}\left(\hat{\theta}_T, q_T - q_0\right)$. Therefore,

$$\mathbb{E}\left[e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0, 0\right)} \mid \hat{\theta}_T, q_T\right] = \omega' e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0, 0\right)},\ \hat{\theta}_0 = \hat{\theta}_T + \mathbf{n}. \tag{IB.10}$$

Second, if $\hat{\theta}_0 < \hat{\theta}_{min}$ or $\hat{\theta}_0 > \hat{\theta}_{max}$, we cannot use Gaussian quadrature and we use the local log linear approximation instead. Note that $\varrho\left(\hat{\theta}_0, 0\right) = \varrho\left(\hat{\theta}_T, 0\right) + \varrho_\theta\left(\hat{\theta}_T, 0\right)\left(\hat{\theta}_0 - \hat{\theta}_T\right)$.

This gives $\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0,0\right)=\varrho\left(\hat{\theta}_T,0\right)-\varrho_\theta\left(\hat{\theta}_T,0\right)\hat{\theta}_T+\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)\hat{\theta}_0$. Therefore,

$$
\begin{aligned}
\mathbb{E}\left[e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0,0\right)}\mid\hat{\theta}_T,q_T\right] &= \mathbb{E}\left[e^{\varrho\left(\hat{\theta}_T,0\right)-\varrho_\theta\left(\hat{\theta}_T,0\right)\hat{\theta}_T}e^{\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)\hat{\theta}_0}\mid\hat{\theta}_T,q_T\right] \\
&= e^{\varrho\left(\hat{\theta}_T,0\right)-\varrho_\theta\left(\hat{\theta}_T,0\right)\hat{\theta}_T}\mathbb{E}\left[e^{\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)\hat{\theta}_0}\mid\hat{\theta}_T,q_T\right], \\
\ln\mathbb{E}\left[e^{\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_0+\varrho\left(\hat{\theta}_0,0\right)}\mid\hat{\theta}_T,q_T\right] &= \varrho\left(\hat{\theta}_T,0\right)-\varrho_\theta\left(\hat{\theta}_T,0\right)\hat{\theta}_T+\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)\hat{\theta}_T \\
&\quad+\frac{1}{2}\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)^2\left(q_T-q_0\right) \\
&= \varrho\left(\hat{\theta}_T,0\right)+\frac{\frac{1}{\psi}-\gamma}{a+\rho}\hat{\theta}_T+\frac{1}{2}\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)^2\left(q_T-q_0\right).
\end{aligned}
$$

Finally, Eq.(IB.9) can be written as

$$
\varrho\left(\hat{\theta}_T,T\right)=\varrho\left(\hat{\theta}_T,0\right)+\frac{1}{2}\left[\left(\frac{\frac{1}{\psi}-\gamma}{a+\rho}+\varrho_\theta\left(\hat{\theta}_T,0\right)\right)^2-\frac{(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)}{(a+\rho)^2}\right]\left(q_T-q_0\right). \quad \text{(IB.11)}
$$

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